# PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

**Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.**

## PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

**2883. Proposed by ROBERT C. COLWELL, Geneva College, Beaver Falls, Pennsylvania.**

Evaluate (*a*) $\int_0^\pi \cos^2\left(\frac{n\pi\cos\theta}{2}\right)d\theta$ and (*b*) $\int_0^\pi \frac{\cos^m\theta d\theta}{\theta^n}$.

**2884. Proposed by E. H. MOORE, University of Chicago.**

Consider an $m \times n$ array $A$ of numbers $a_{st}$ and an $n \times m$ array $B$ of numbers $b_{ts}$. Show that the system of $mn$ equations:

$$\sum_{tu} a_{st}b_{tu}a_{uv} = 0 \qquad (sv),$$

implies the equation:

$$\sum_{ts} a_{st}b_{ts} = 0.$$

The suffixes $s$, $u$ have the range 1, 2, $\cdots$, $m$ and the suffixes $t$, $v$ have the range 1, 2, $\cdots$, $n$.

**2885.**

If $A, B, C, X, Y$ are given collinear points, construct $Z$ so that $\{ABCX\}+\{ABCY\}=\{ABCZ\}$, where $\{ABCX\}$ denotes the cross-ratio of the points $A, B, C, X$. [From *the Math. Tripos Exam.*, Cambridge, Eng., 1905.]

**2886. Proposed by S. A. COREY, Des Moines, Iowa.**

If, in considering the purchase of bonds in the open market, $x$ = interest yield to investor; $i$ = interest rate named in bond; $t$ = time in years to maturity; and $q$ = quoted market price, the relation between these quantities is expressed by the equation, $xq - (x - i)(1 + x)^{-t} - i = 0$. Find value of $x$.

Also show that, if $a = i/q + (1 - q)/qt$, we may, in practice, safely assume that very nearly,

$$x = \frac{at(a - i) + i[(1 + a)^{t+1} - (1 + a)]}{q(1 + a)^{t+1} + t(a - i) - (1 + a)}.$$

**2887. Proposed by the late L. G. WELD.**

A carpenter's square moves with its outer edges in contact with two round pegs of given equal diameters. Define the locus of the "heel" of the square.

**2888. Proposed by J. A. BULLARD, U. S. Naval Academy.**

Discuss the surface, $x^{1/2} + y^{1/2} + z^{1/2} = a^{1/2}$, by means of plane sections. (Cf. problem 2846, *1920*, 326.)

**2889. Proposed by W. D. LAMBERT, U. S. Coast and Geodetic Survey.**

Evaluate, $\int_c^1 \frac{\cos^{-1} x dx}{\sqrt{x^2 - c^2}}$, where $0 < c < 1$.